Observation of harmonic generation and nonlinear coupling in the collective dynamics of a Bose condensate

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Since the first observations of Bose condensates [1] the study of collective dynamics and the spectrum of their excitations has played a crucial role in the understanding of these new mesoscopic systems. Measurements on the lowest energy modes [2, 3] verified that the Nonlinear Schrödinger Equation (NLSE) gives a very accurate prediction of the frequencies of the measured collective modes. In our experiment we could directly observe effects arising from the nonlinear term in this equation. We used a modified design of a TOP trap for which the trap anisotropy ($\lambda = \omega_z/\omega_r$, where ω_z is the axial and ω_r the radial trap frequency) can be tuned continuously over a range of values where nonlinear coupling is very large. This region is centred around a resonance between the $m = 0$ low-lying and the $m = 0$ high-lying mode (at $\lambda \approx 1.95$). At this resonance two quanta of excitation of the low-lying (fundamental) mode are converted into one quantum of excitation of the high-lying mode, i.e. the second harmonic of the low-lying mode is created. We excited the low-lying mode by modulating the TOP-field amplitude sinusoidally at the frequency of the low-lying mode for 8 cycles. Close to resonance we observed the occurrence of the second harmonic frequency in the oscillations of the condensate due to the nonlinear coupling between the high and the low-lying mode. On resonance we found that the fundmental mode is strongly suppressed and most of the energy had been transferred into the second harmonic mode in agreement to theoretical predictions [4]. We also observed significant negative frequency shifts of the fundamental on the higher side ($\lambda > 1.95$) and positive frequency shifts on the lower side ($\lambda < 1.95$) of the resonance. Frequency shifts are predicted to be proportional to the square of the excitation response amplitude to first approximation, i.e. $\Delta\omega/\omega_0 = A^2\delta(\lambda)$, where A is the response amplitude, $\delta(\lambda)$ is a nonlinearity factor which varies according to the trap anisotropy λ and ω_0 is the zero amplitude response frequency. The nonlinearity factor δ determines the magnitude and sign of the frequency shifts. A theoretical plot[5] is given in Fig.1a for the nonlinearity around the resonance between the two modes. There is good agreement with the measured frequency shifts which are plotted against the trap anisotropy in Fig.1b and the mode frequencies clearly follow a dispersive curve. The solid line is a theoretical prediction[6] for the zero amplitude mode frequencies for the number of atoms (typically 2×10^4) and trap anisotropies in our experiment. This curve agrees very well with our data far from resonance where the nonlinearity δ and hence the frequency shifts are very small. The mode frequencies in the hydrodynamic limit[7] are plotted for comparison (dotted line). Future improvements and modifications of our experiment will allow us to study additional nonlinear phenomena such as the collapse and revival of oscillations and the onset of chaos for stronger driving.

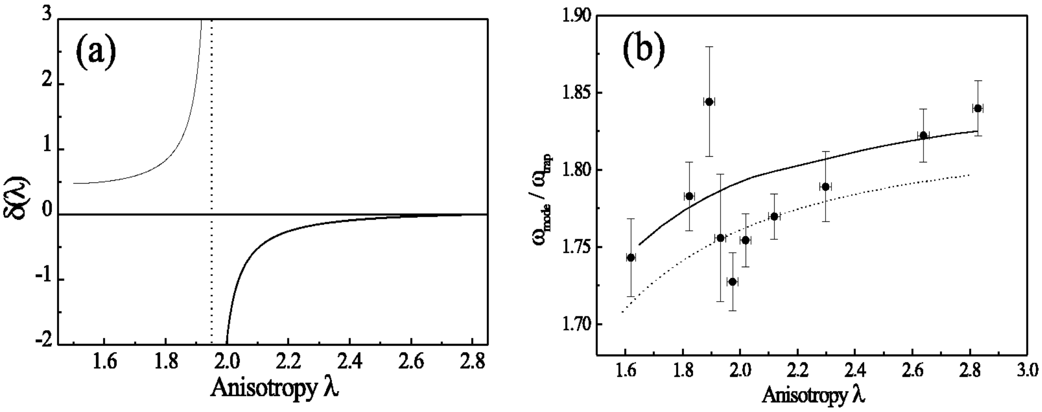

Figure 1: (a) The nonlinearity $\delta(\lambda)$ is plotted against the trap anisotropy λ. (b) The measured low-lying mode frequencies normalized by the corresponding radial trap frequencies are plotted against the anisotropy. The solid line is a finite-number theoretical prediction[6]. The dotted line shows the theoretical prediction in the hydrodynamic limit for comparison.

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